A DIAGNOSTIC STUDY OF MID-TROPOSPHERIC DEVELOPMENT

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[Manuscript received November 9, 1960; revised January 11, 1961]

ABSTRACT

An equation for vertical velocity is applied to data from 850, 500, and 200 mb. for calculations of vertical velocity and divergence in special cases characterized by pronounced failures of barotropic forecasts. The results of the calculations show that a non-development situation was adequately described by the equivalent-barotropic picture of a single quasi-horizontal surface of non-divergence. The onset of mid-tropospheric development, as shown by the appearance of large errors of the barotropic forecast, was characterized by the appearance of a double surface of non-divergence, with a deep mid-tropospheric convergence layer in the vicinity of the trough line. This picture of development is confirmed by a second case study.

The appearance of the mid-tropospheric convergence layer is related to the low-level cold push, the tilt of the flow patterns with height, and the high-level jet stream, confirming synoptic studies by J. J. George, H. Riehl, and others.

1. INTRODUCTION

In considering the problem of baroclinic development, usually associated with cyclogenesis, one is often led to consider the various theoretical studies of baroclinic instability. Such studies for two-parameter prediction models yield quantitative instability criteria. However, when applied to atmospheric data, two-parameter prediction models produce forecast failures strikingly similar to those of barotropic forecasts, especially in cases of pronounced baroclinic development.

In considering the capabilities of a two-parameter model which contains representations of vertical shear and vertical velocity, from which vertical momentum advection can be computed, it is difficult to imagine one which does not contain, by implication at least, a quasi-horizontal surface of non-divergence. It therefore seems likely that a feature of the atmospheric circulations not found in two-parameter models, geostrophic, balanced, or primitive equation models, might be of considerable importance in the development process. One such feature which should be considered is the mid-tropospheric divergence, which can be computed from an atmospheric model having three or more parameters in the vertical.

This study will attempt to throw some light on the development process by means of computing vertical velocities and various derived fields by means of a three-level model of the atmosphere. For the purpose of this study, development can be considered as the processes which lead to error in the barotropic forecast (over areas of good data). We shall also exclude special mountain and friction effects from this study. In order to bring out more clearly the development processes, the actual events will be described in terms of the errors of the barotropic forecasts. Vertical motion, divergence, and vertical momentum advection can then be discussed in terms of these errors.

2. THE VERTICAL VELOCITY EQUATION

The vertical velocity equation is obtained in the usual method from the vorticity equation and the adiabatic equation written in the following form:

$$\frac{\partial\zeta}{\partial t}+\mathbf{V}\cdot\nabla\eta-\eta_0\frac{\partial\omega}{\partial p}=0 \tag{1}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial p}\right)+\mathbf{V}\cdot\nabla\frac{\partial\phi}{\partial p}+\omega\sigma=0. \tag{2}$$

In (1) and (2) $\mathbf{V}$ is the horizontal wind; ζ is the relative vorticity; η is the absolute vorticity; η_0 is the absolute vorticity at 500 mb. which can also be used to represent a vertical average of η; $\omega=dp/dt$, the vertical velocity; ϕ is the geopotential; and σ is a measure of the static stability given by $\sigma=-\alpha\partial\ln\theta/\partial p$, where α is specific volume and θ is potential temperature.

We now assume that for the purposes of calculating horizontal advection the horizontal wind is non-divergent, and can be represented by a streamfunction, i. e., $\mathbf{V}=\mathbf{k}\times\nabla\psi$. In place of the usual formulation of the geostrophic approximation necessary to obtain an equation for ω, we shall make an approximation to the temperature field that $\partial\phi/\partial p=f\partial\psi/\partial p$. Introduction of these approximations into (1) and (2) leads to the ω-equation:

$$\nabla^2\omega+\frac{f\eta_0}{\sigma}\frac{\partial^2\omega}{\partial p^2}=\frac{f}{\sigma}\left[\frac{\partial}{\partial p}(\mathbf{V}\cdot\nabla\eta)-\nabla^2\left(\mathbf{V}\cdot\nabla\frac{\partial\psi}{\partial p}\right)\right]. \tag{3}$$

This involves the assumption that horizontal variations of static stability can be neglected, although σ is permitted to vary in the vertical. The appearance of $f\eta_0$ as a coefficient of the second term in equation (3) was required for special reasons not relevant to this study. However, different versions of equation (3) were used in calculations in which (a) static stability varied freely, involving extra terms in σ, and (b) f_0^2 (at 45° N.) replaced $f\eta_0$. The results which applied to this study were not significantly affected by these changes.

The finite difference system in the vertical consisted of using data from 850, 500, and 200 mb. An interpolation was made enabling the use of 800, 500, and 200 mb. in the calculations. In computing finite differences at the low levels, the variable height of the ground was taken into consideration. The upper and lower boundary conditions consisted of $\omega=0$ at $p=0$ and $\omega=\mathbf{V}_g\cdot\nabla p_g$ at the lower boundary p_g, the standard atmosphere pressure at the variable height of the ground.

For finite differences in the horizontal the Joint Numerical Weather Prediction Unit's (JNWP) grid on a polar stereographic map having a mesh length of 381 km. at 60° N. was used. In the computation of Laplacians and Jacobians, a consistent system was used in which the components u and v of the wind were taken over a double mesh length. The calculations were made on the IBM 704 computer.

Before proceeding to the case studies, let us consider certain implications of the vertical velocity equation. For this purpose we can simplify equation (3) to the form:

$$\frac{\partial^2\omega}{\partial p^2}-A\omega=F, \tag{4}$$

where $A=k^2\sigma/f\eta_0$, k being a horizontal wave number. The right-hand side of the equation is represented as the forcing function, F, determined by the vertical variation of vorticity advection and the Laplacian of the temperature advection. In a quasi-geostrophic model, n-1 values of F can be obtained from n parameters in the vertical. The usual two-parameter prediction model therefore contains sufficient information for only one value of F in the vertical. If we then take F as independent of pressure, and apply as boundary conditions $\omega=0$ at 1000 mb. and at 0 mb., the ω-profile is symmetric about 500 mb., where a surface of non-divergence is found. This is represented by figure 1a. Of course it would be possible, in the absence of other than climatological information, to make an assertion regarding the variation of F with pressure in such a model. The only effect of this would be to change the pressure of the non-divergent surface. However, a continuous non-divergent surface corresponding with an isobaric surface would still exist.

Suppose instead that we consider a model with sufficient parameters in the vertical (minimum of three) to provide information on the slope of the vertical F-profile. Figures 1b and 1c represent the shapes of the ω-profile associated with two simple distributions of F. It is clear that if the slope of the F-profile changes sign from one place to

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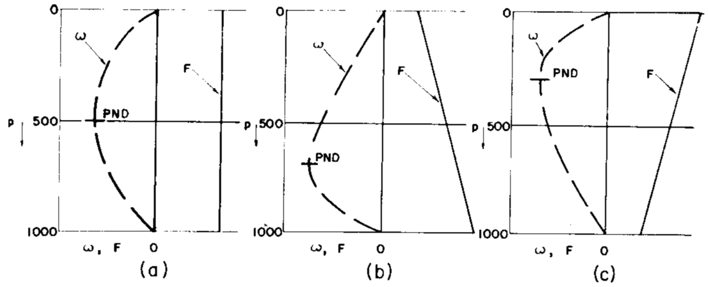

FIGURE 1.—Three vertical profiles, (a), (b), and (c), of forcing function, F, and vertical velocity, ω, as a function of pressure p. PND refers to the pressure at which no horizontal divergence exists.

another significant areas of mid-tropospheric convergence or divergence may occur. An atmospheric model with three or more levels is thus suitable for calculation of mid-tropospheric divergence. Although detailed numerical calculations will be necessary before the question of the number of levels required for adequate vertical resolution can be settled, it is clear that three levels represents the minimum number required if mid-tropospheric divergence is an important baroclinic phenomenon.

3. CASE STUDIES

(I). JANUARY 21–22, 1959

On January 22, 1959, a notable cyclogenesis was observed over North America. A sea level cyclone passed across the Great Lakes and travelled into Labrador, deepening rapidly. Of special interest was the fact that large errors rather suddenly appeared in even short-range barotropic forecasts.

The development was preceded by the motion of a 500-mb. trough through the central United States. Figure 2a shows that 24 hours in advance of the development the atmosphere was behaving barotropically, as shown by the small errors of the barotropic forecasts. The cross-section of horizontal divergence for the same time (fig. 2b) shows the familiar equivalent barotropic pattern of a quasi-horizontal surface of non-divergence near 500 mb. with compensating convergence and divergence patterns above and below the surface. Some distortion of the pattern is found over the mountains as a consequence of the ascent of the air to the west and the descent to the east of the highest terrain.

Figure 3 shows a more complicated situation by 1200 GMT of January 21. This could be regarded as a period of transition from an equivalent barotropic situation to a baroclinic situation. The errors of the barotropic forecast for the subsequent 12 hours increased to amounts which were not negligible. The cross-section (fig. 3b) shows a split of the surface of non-divergence in the vicinity of the trough line. The signs of the convergence and divergence patterns in mid-troposphere are consistent with the errors observed in the barotropic forecast.

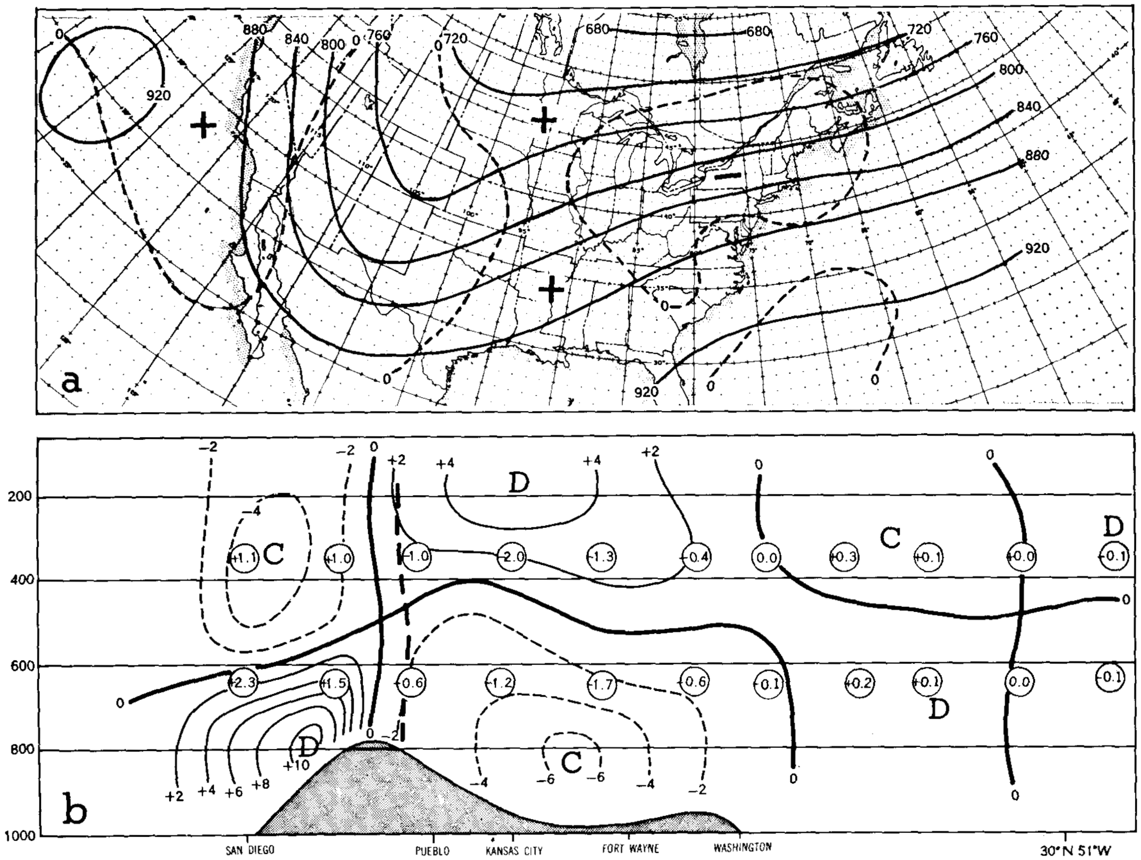

FIGURE 2.—(a) Error in decafeet (dashed lines) of the 12-hr. barotropic forecast from 0000 GMT, January 21, 1959, superimposed on the 500-mb. contours (solid lines) for that time. (b) Cross-section of horizontal divergence in units of 10^{-6} sec.$^{-1}$. The numbers in circles are ω in units of 10^{-3} mb. sec.$^{-1}$. Heavy dashed line is the trough line.

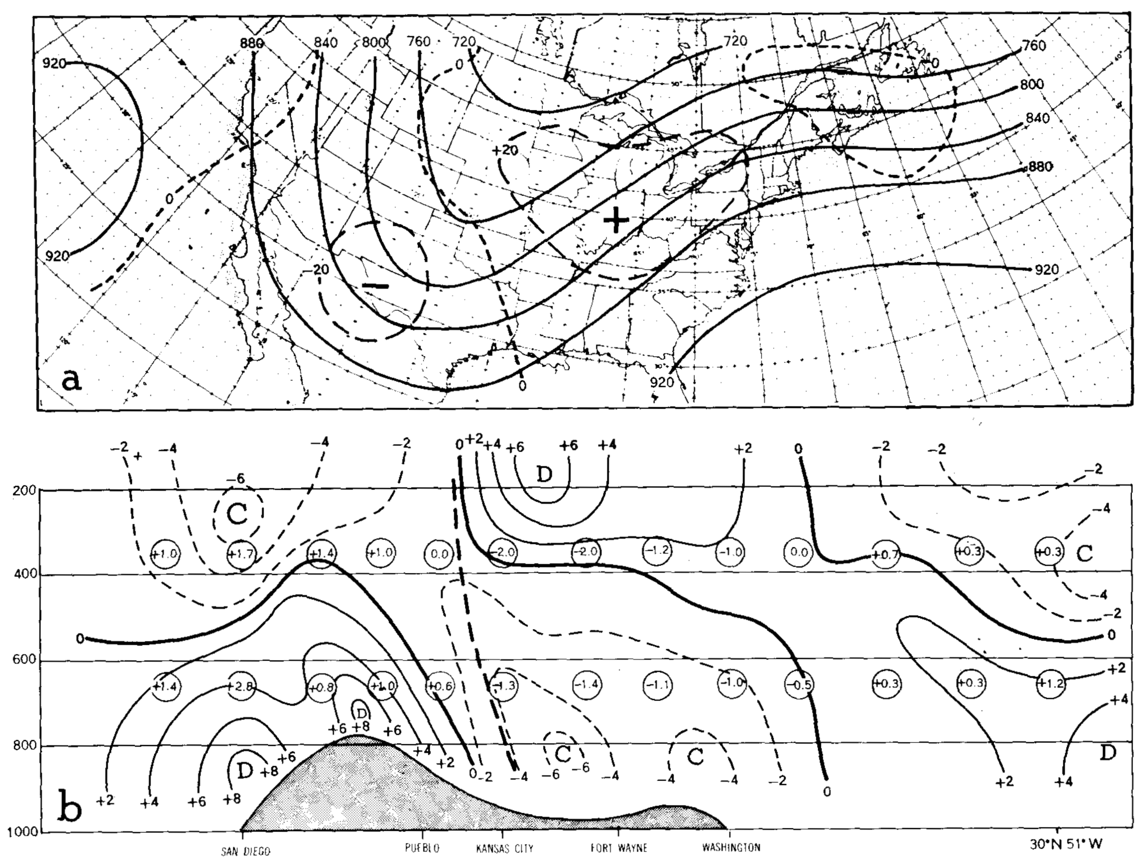

FIGURE 3.—Same as figure 2 except for 1200 GMT, January 21, 1959.

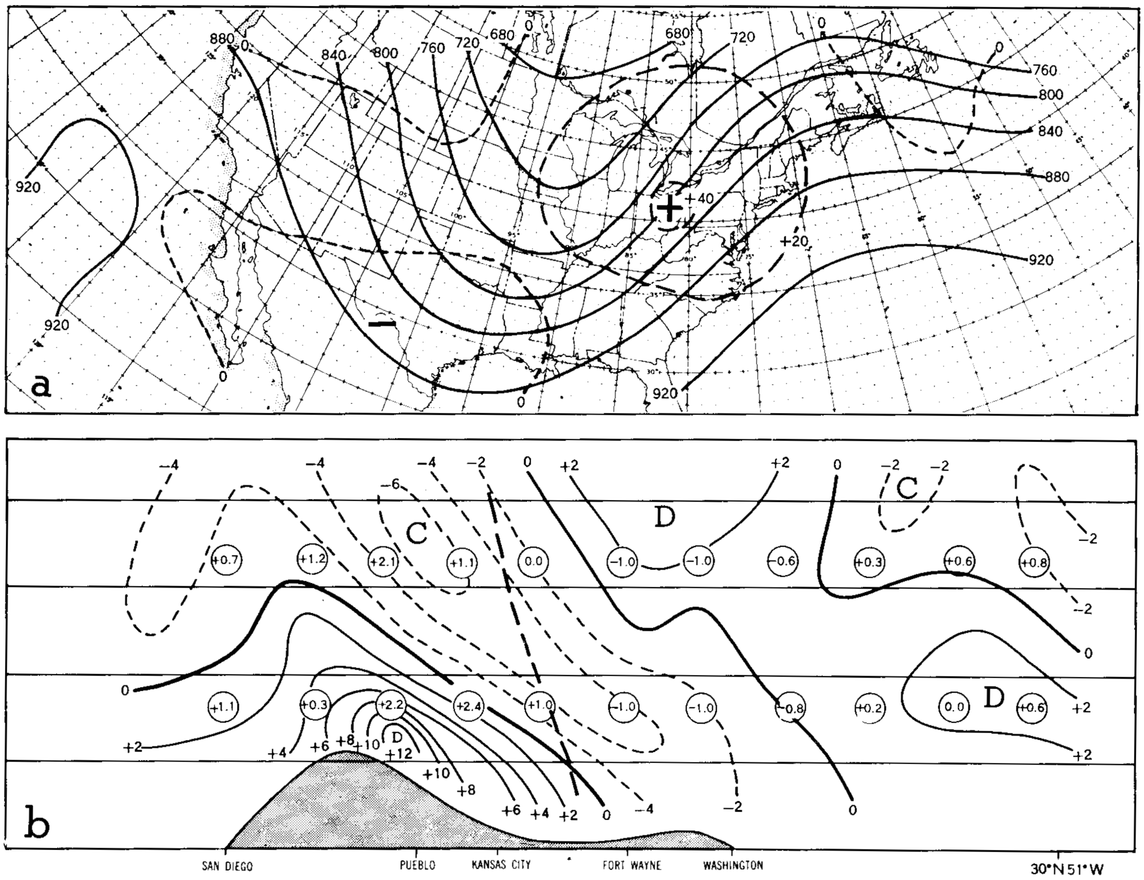

FIGURE 4.—Same as figure 2 except for 0000 GMT, January 22, 1959.

In figure 4 the development is shown proceeding in strength. The error of the barotropic forecast over the eastern United States was extreme. The cross-section (fig. 4b) shows a most significant change from conditions 24 hours earlier. In the vicinity of the trough line there was a double surface of non-divergence, one at high levels and one at low levels, with a deep layer of convergence in mid-troposphere. This cross-section of a developing trough resembles similar cross-sections obtained by Charney [3] and by Hinkelmann [6] from idealized data, as well as those computed from real data by Bundgaard [2] and by Fleagle [4].

Having the vertical velocities, one can compute the effects at 500 mb. of the vertical advection of momentum. The tendency of the 500-mb. height arising from the vertical momentum advection is shown in figure 5. It can be seen that in this situation this effect is generally of a sign to reduce 500-mb. error, but can account at the most for about 20 percent of the error.

In considering that vertical velocities of 10^{-3} mb./sec, or more over large areas are relatively common in winter, one sees that vertical displacements of 100 mb./day should result. If the vertical velocity is downward in a trough line, this could result in a strengthening of the flow

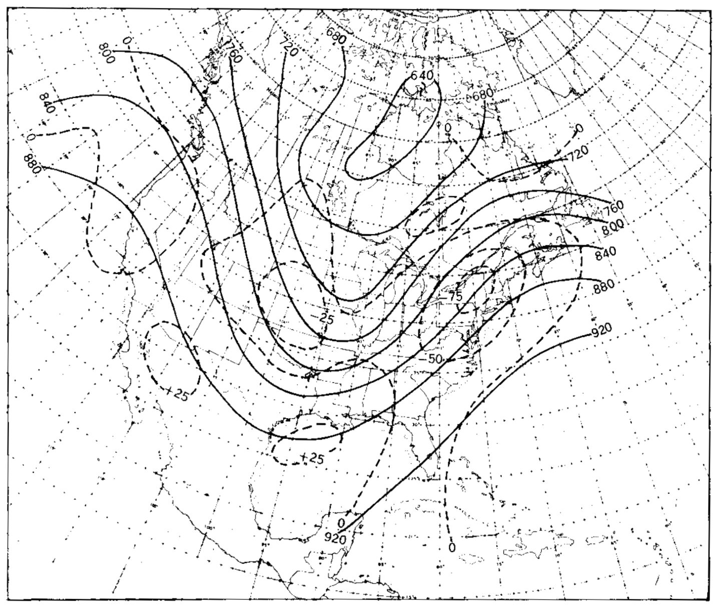

FIGURE 5.—Contribution of the vertical advection of momentum to the 500-mb. height tendency (dashed lines; units of ft. per 12 hr.) for 0000 GMT January 22, 1959, superimposed on the 500-mb. contours (solid lines) for that time.

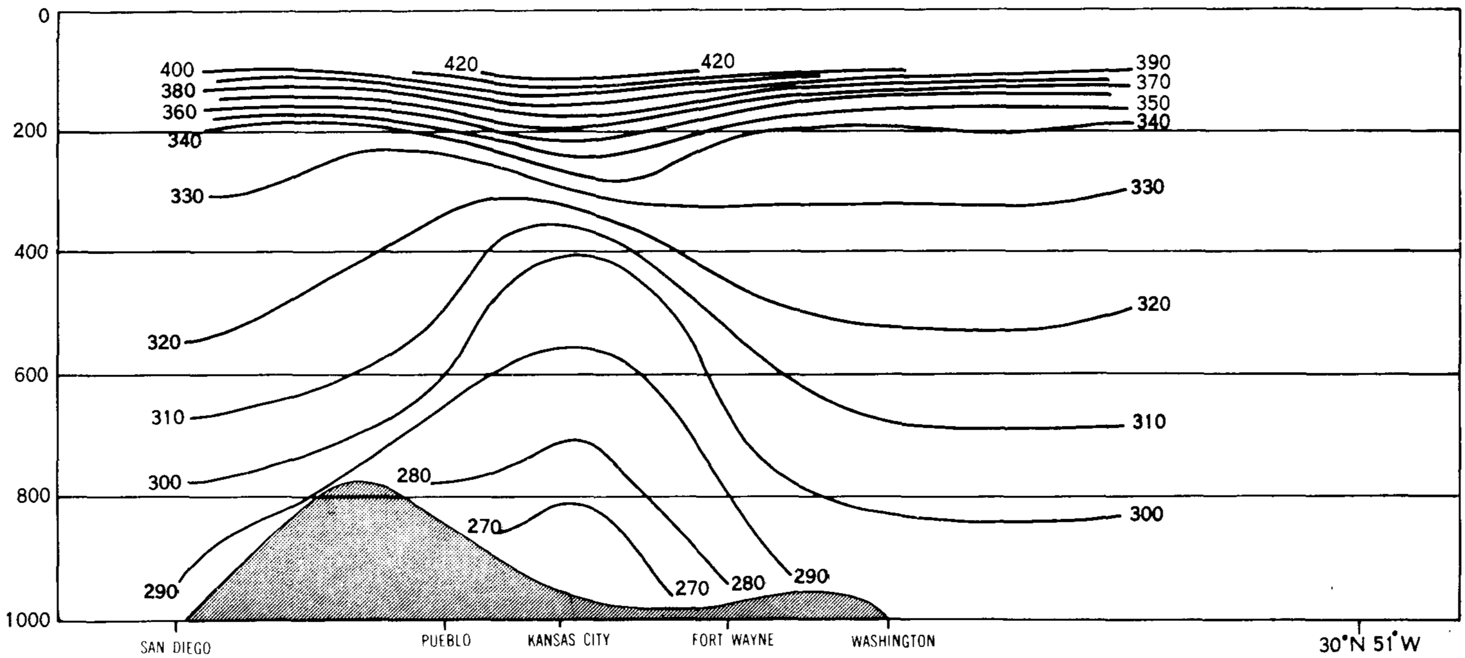

FIGURE 6.—Cross-section for 0000 GMT, January 22, 1959, showing potential temperature in the same vertical plane as in figure 4b.

through a deep layer by 20 percent in 24 hours, an amount which is by no means negligible.

In figure 6, the intersections of the isentropic surfaces with the cross-section are shown. These show the pronounced cold dome, and cold push, to be a feature of the lower two-thirds of the atmosphere. Above this we find a warm push, associated with the low warm stratosphere.

(II). DECEMBER 6, 1959

On December 6, 1959, a cyclogenesis occurred along the eastern United States coast. Figures 7 and 8 show the extreme error of the barotropic forecast over the southeastern States as the low-level cyclogenesis took place. The computed vertical velocities at 650 and 350 mb., together with the computed 500-mb. divergence, are shown in figures 9, 10, and 11. Examining the areas of the southeastern States, we can see that a large area of 500-mb. convergence was associated with two separate features. Ahead of the trough the ascending motion was stronger at high levels than at low levels, and in the trough line the descending motion was stronger at low levels. A 500-mb. divergence area is found just behind the trough line, where the descent was strongest at high levels. The 500-mb. divergence pattern of figure 10 can be compared with that of figure 6b in the paper by Brown and Neilon [1], who obtained their map by assuming that the entire error of the barotropic forecast was a result of the 500-mb. divergence pattern. Although there are considerable differences in details, there is a good resemblance of the major features.

Another view of this can be obtained from cross-sections running just north of the main centers of activity. The orientation of the cross-section (fig. 12) was chosen to coincide with that presented in the following paper by Brown and Neilon. This gives the reader an opportunity to check two completely independent determinations of vertical velocity and divergence. The cross-section of divergence (fig. 12b) shows a pattern in the developing trough similar to that of figure 4. The vertical velocity cross-section (fig. 12a) shows that in this case the convergence associated with the trough was associated mainly with a strong ascending motion at high levels. This relatively strong high-level ascent, as well as the strong descent behind the trough, was a consequence of a strong 200-mb. jet stream in strongly curved flow through the trough. If there had been no phase shifts in the vertical, this would merely have resulted in a faster eastward displacement of the 500-mb. trough than given by the barotropic forecast, since at 500 mb. there would have been convergence ahead of and divergence behind the trough. However, the phase shift in the vertical resulted in an extension of the 500-mb. convergence area into the trough line as well as an enlargement of the convergence area. This conclusion is in agreement with the results of Wiin-Nielsen [9] who showed that the lagging of the temperature field behind the flow leads to a mid-tropospheric tendency for deepening in the trough line.

4. GENERAL REMARKS

A feature of both cases was the tendency for the sinking of cold air to be at a maximum in the lower levels. A well-known characteristic of strong cold-air outbreaks is the tendency for the maximum cold-air advection to occur in the lowest half of the atmosphere, with a low, warm stratosphere prominent in the higher levels.

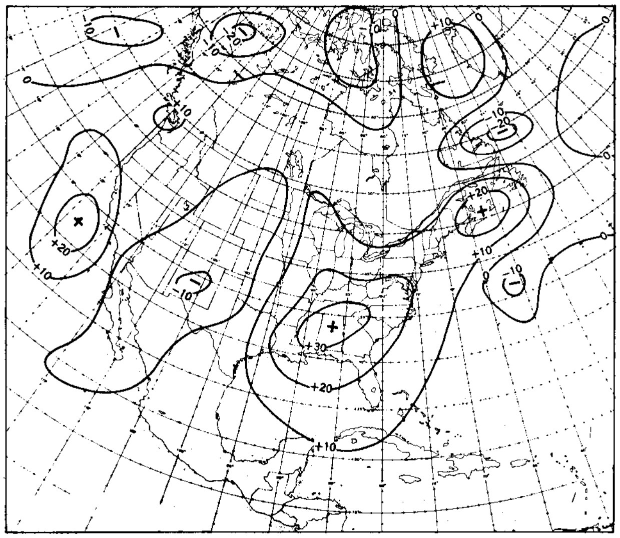

FIGURE 7.—Error in decafeet of 12-hr. barotropic forecast from 0000 GMT December 6, 1959.

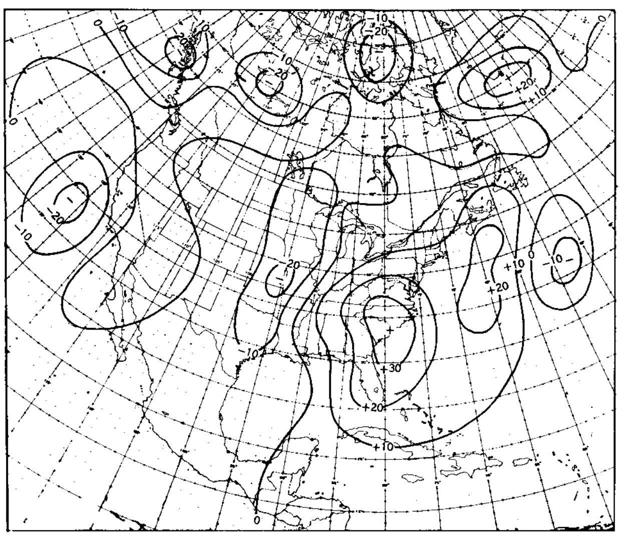

FIGURE 8.—Error in decafeet of 12-hr. barotropic forecast from 1200 GMT December 6, 1959.

According to the analysis of the ω-equation presented earlier, such cold-air outbreaks therefore ought to be characterized regularly by a very strong low-level divergence, a relatively low surface of non-divergence, and a layer of mid- and upper-tropospheric convergence. In such cases, failures of barotropic forecasts are often observed. Cyclogenesis is also frequently seen under such conditions. In support of this diagnosis we can cite the cyclogenesis study by George and collaborators [5], who found the strong low-level cold-air push to be a reliable precursor of cyclogenesis. They found the 850-mb. chart to be most useful in recognition of the low-level cold push. J. Austin* has pointed also to the occurrence of the abnormally low warm stratosphere during the initial stages of cyclogenesis.

* In lectures at two American Meteorological Society meetings at Washington, D.C. in 1954 and 1955.

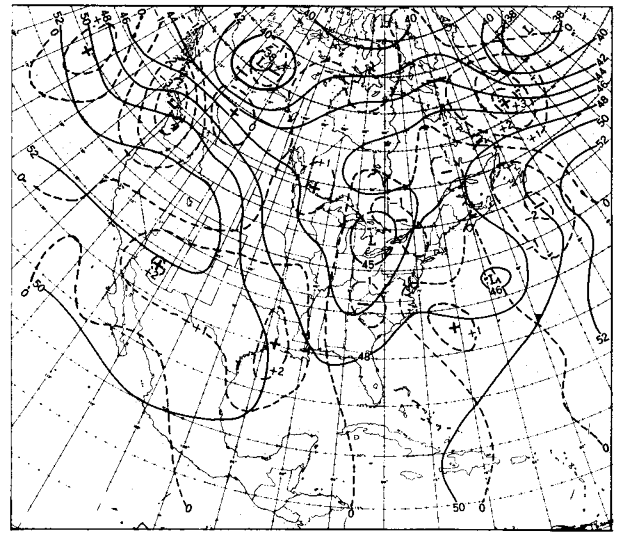

FIGURE 9.—850-mb. contours (solid lines) and 650-mb. ω (dashed lines) in units of 10^{-3} mb. sec.$^{-1}$ for 1200 GMT, December 6, 1959.

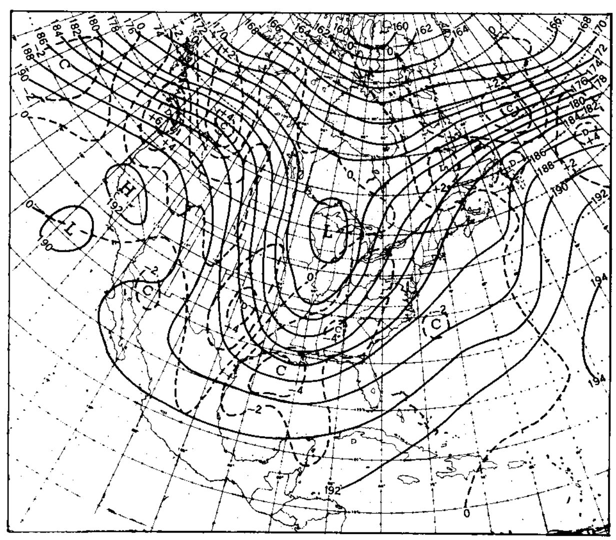

FIGURE 10.—500-mb. contours (solid lines) and divergence (dashed lines) in units of 10^{-6} sec.$^{-1}$ for 1200 GMT, December 6, 1959.

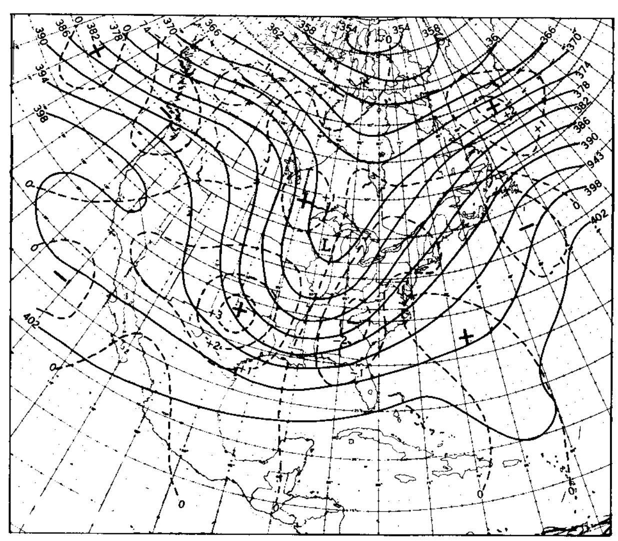

FIGURE 11.—200-mb. contours (solid lines) and 350-mb. ω (dashed lines) in units of 10^{-3} mb. sec.$^{-1}$ for 1200 GMT, December 6, 1959.

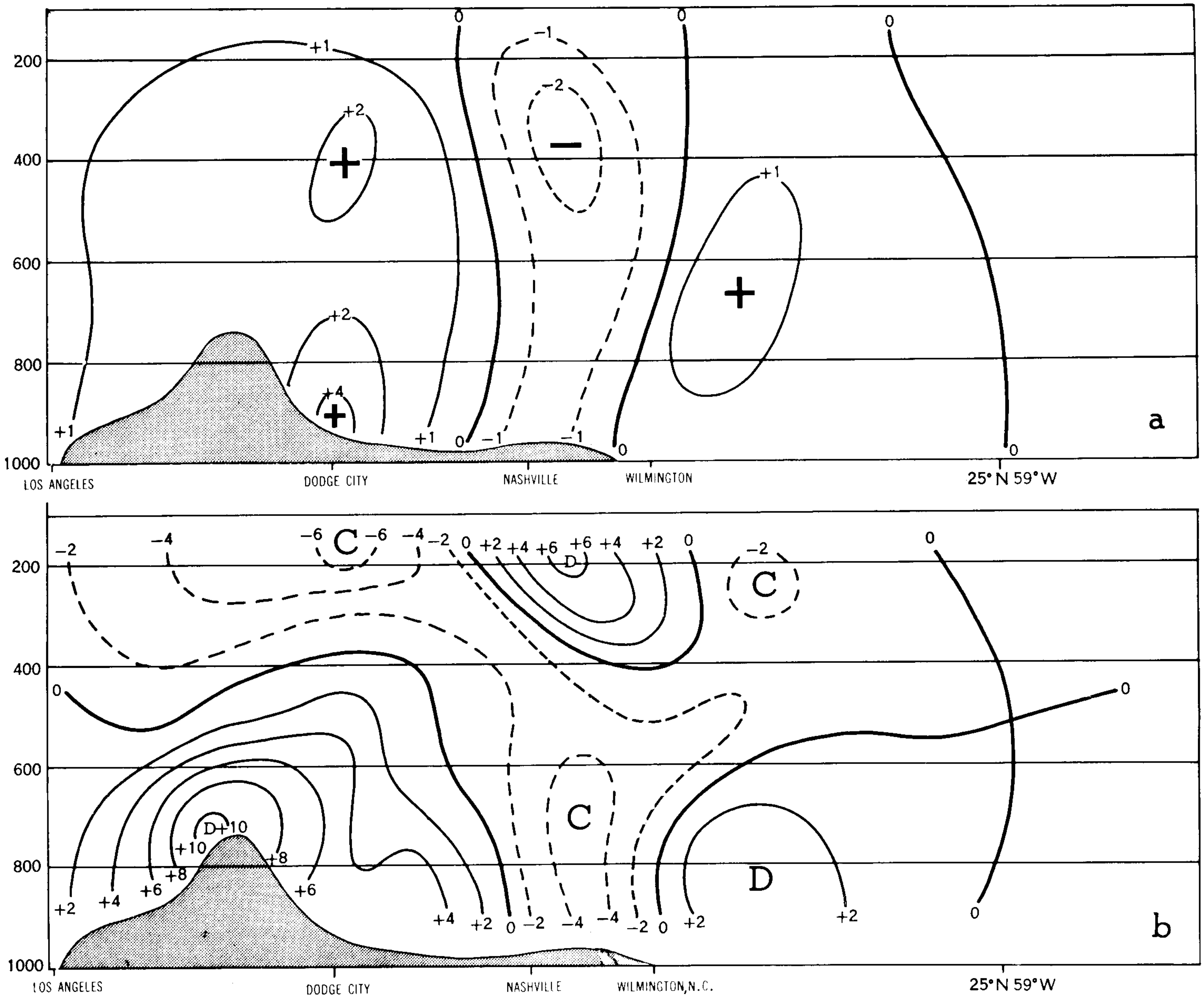

Figure 12.—Cross-section (a) vertical velocity (10^{-3} mb. sec.$^{-1}$) and (b) divergence (10^{-6} sec.$^{-1}$), for 1200 gmt, December 6, 1959.

Wiin-Nielsen [9] showed in his 3-level analysis of a model disturbance that an upward increase in vertical wind shear leads to a pattern of 500-mb. divergence which tends to speed up systems (convergence ahead of a trough, divergence behind), as compared with barotropic motion. Taba's [8] typical vertical profiles of the subtropical jet stream show the normal existence of such conditions. It is a common experience at the JNWP Unit that the barotropic forecasts move features too slowly in the areas occupied by a subtropical jet stream. If a subtropical jet stream is found over the forward side of a trough, as in the schematic representation of figure 13, the resulting mid-tropospheric convergence accentuates the development. Here we should point out that Riehl [7] observed this association in 1947. His paper contains special mention of the upward increase of shear under the high-level jet stream, in association with the occurrence of cyclogenesis.

5. CONCLUSIONS

The equivalent barotropic representation of the atmosphere, with a quasi-horizontal surface of non-divergence near 500 mb. separating divergence patterns of opposite sign above and below, was a realistic representation of actual conditions during a period preceding cyclogenesis. However, during two different cyclogenetic situations, the equivalent barotropic idea became invalid. During these development periods a double non-divergent surface appeared in the vicinity of the developing trough.

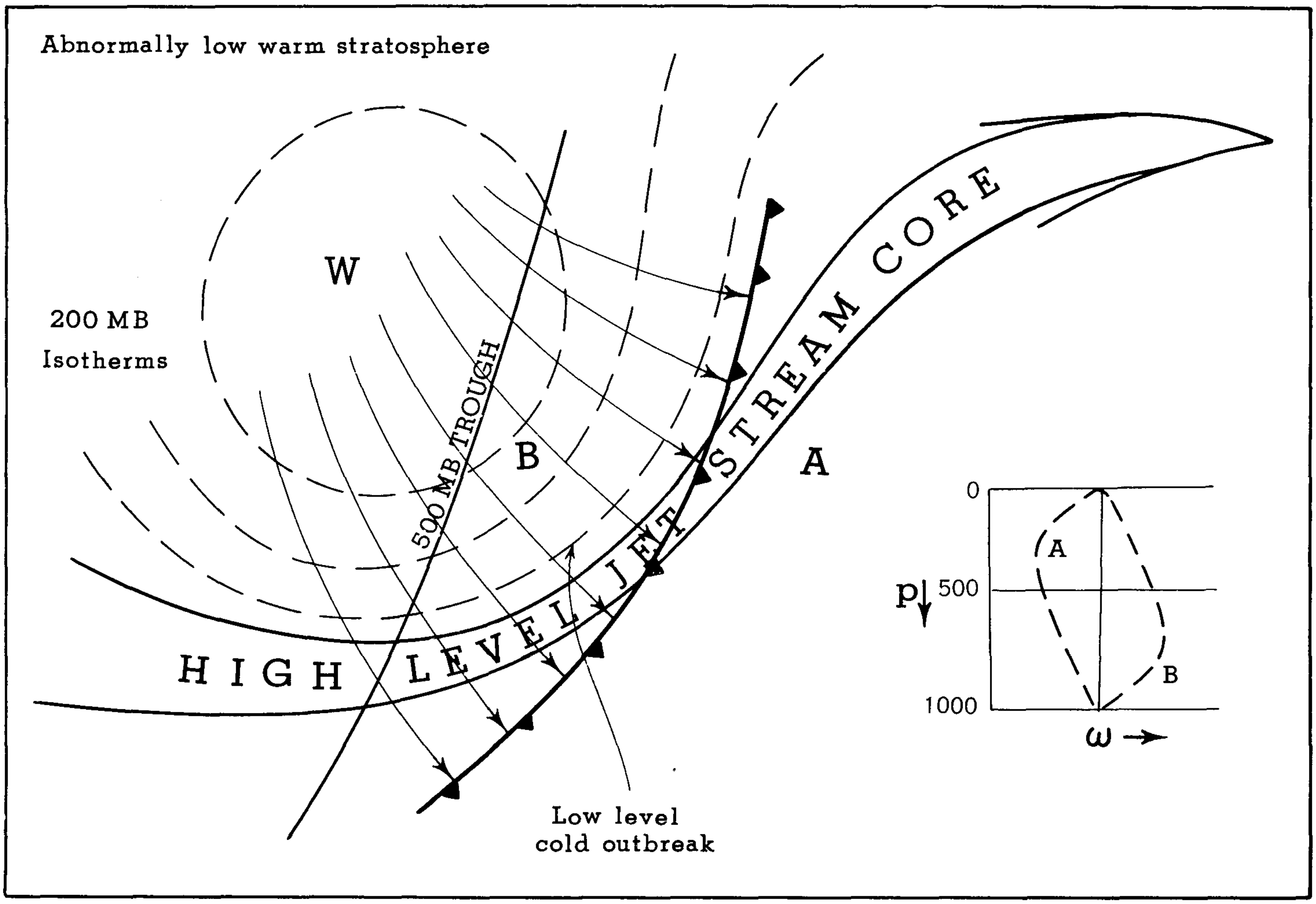

FIGURE 13.—Schematic representation of factors associated with mid-tropospheric development.

Between upper and lower divergence regions a deep mid-tropospheric convergence layer appeared. The invalidation of the equivalent barotropic representation was further demonstrated by the appearance of large errors in the barotropic forecasts.

The particular atmospheric features responsible for this mid-tropospheric convergence area near the developing trough line were the following:

(1) The low-atmospheric strong cold-air advection in and behind the low-level trough surmounted by high-level warm air advection. This led to a vertically asymmetric ω-profile, with low-level sinking and mid-tropospheric convergence.

(2) The normal phase shift with height of the system, in which the sinking cold air is brought in at lower levels under the upper cyclonic system. This accentuates the effects mentioned under (1) above.

(3) The participation of a strong high-tropospheric jet stream in the strongly curved upper flow. This accentuates the pre-trough convergence in mid-troposphere.

These features are represented in figure 13, in which the low-level cold push, the high-level jet stream, and the phase shift with height are indicated. The inset ω-profiles represent conditions at the locations A and B. One cannot expect that all of these contributing factors will be present in each case of mid-tropospheric development, but the errors of the barotropic forecasts will reach the largest values when all are present in strength.

The most serious limitations on the accuracy of the calculations presented here are the use of the geostrophic approximation, and the lack of a better vertical resolution of the atmosphere. It is just in development situations where the vertical velocities and ageostrophic motions reach their greatest magnitudes. At the same time the smaller-scale horizontal motions become of greater importance to the developmental process. It is therefore, a very large step from diagnosis to prognosis. However, in deriving an ω-equation of the general type used in this study it is necessary to state some kind of wind law. In view of the well-known sensitivity of numerical predictions to the wind law used, it seems that a generalization of the wind law to be used in the ω-equation to that of the balance equation might be of sufficient interest to warrant recoding the machine calculations.

The problem of vertical resolution of the calculations is aggravated by the presence of a tropopause of variable height. If the vertical wind shears above and below the tropopause are each relatively invariant (but of opposite sign) serious errors can be introduced into the estimates of vertical wind derivatives from streamfunctions at widely separated levels on opposite sides of the tropopause. Further experiments to study this question seem to be in order.

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